

# Spin and orbital Kondo effect in electrostatically coupled quantum dots

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The spin polarization of conductance in a system of two capacitively coupled quantum dots is studied in the Kondo regime by the equation of motion method. In the case of orbital degeneracy in one spin channel, the system can operate as a spin filter.

Key words: *spin-polarized transport; nanoscopic system; Kondo effect*

## 1. Introduction

Spin-dependent coherent electronic transport recently attracts great interest due to its potential applications in reprogrammable logic devices [1] and quantum computers [2]. One of the challenging problems for such applications is to obtain sufficient control over spin dynamics in nanostructures. In the following, we address this issue and propose a double dot setup that can operate as a spin filter. Our proposal utilizes spin-dependent orbital degeneracy and the resulting orbital Kondo effect as a mechanism for spin filtering. In this case, the orbital quantum number is conserved during tunnelling and orbital degrees of freedom come into play as pseudo-spins [3]. The orbital Kondo effect has been observed in semiconductor double quantum dots [4, 5] and carbon nanotubes [6].

## 2. Model

We discuss a system of two capacitively coupled quantum dots (DQD) placed in a magnetic field. Each of the dots is coupled to a separate pair of leads. The corresponding Hamiltonian reads:

$$H = \sum_{kri\sigma} \varepsilon_{kri\sigma} c_{kri\sigma}^\dagger c_{kri\sigma} + \sum_{i\sigma} \varepsilon_{i\sigma} c_{i\sigma}^\dagger c_{i\sigma} + \sum_i U_i n_{i+} n_{i-} + U \sum_{\sigma\sigma'} n_{1\sigma} n_{2\sigma'} + \sum_{kri\sigma} t_{ri} (c_{kri\sigma}^\dagger c_{i\sigma} + cc) \quad (1)$$

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where  $\varepsilon_{i\sigma} = \varepsilon_i + \mathbf{g}_\sigma \hbar$  (we set  $|e| = \mu_B = k_B = 1$ ). The first term describes the electrons in the electrodes ( $i = 1, 2, r = L, R$ ), the second represents field-dependent site energies, the third and fourth – account for intra and inter-Coulomb interactions, and the last one describes tunnelling. Assuming quasi-elastic transport, for which the current conservation rule is fulfilled for any  $\omega$ , one obtains an expression for the current  $I = \sum_{i\sigma} I_{i\sigma}$ , within the Keldysh

formalism, in the form [7] :

$$I_{i\sigma} = \frac{e}{\hbar} \int d\omega \frac{\Gamma_{iL\sigma}(\omega)\Gamma_{iR\sigma}(\omega)}{\Gamma_{iL\sigma}(\omega) + \Gamma_{iR\sigma}(\omega)} [f_{iL}(\omega) - f_{iR}(\omega)] \rho_{i\sigma}(\omega) \quad (2)$$

where  $\rho_{i\sigma}(\omega) = (-1/\pi)\text{Im} G_{i\sigma}^r(\omega)$ ,  $G_{i\sigma}$  denotes Green's function of the dot, and  $f_{ir}$  are the Fermi distribution functions of the electrodes. We restrict ourselves here to the case of equal couplings to the dots, i.e.  $t_{ri} \equiv t$ . Bare Green's functions of the electrodes,

$$\mathbf{g}_{r\sigma} = \sum_k \mathbf{g}_{kr\sigma} = \sum_k \frac{1}{\omega - \varepsilon_{kr\sigma}} \equiv g, \quad \text{are taken in the form } g = -i\pi\rho_0, \quad \text{where } \rho_0 = 1/2D \text{ is the}$$

assumed constant density of states for  $|\varepsilon| < D$ , and  $D$  is the half of the bandwidth of electrons in the electrodes. Consequently, the elastic couplings to the electrodes are energy independent,  $\Gamma_{ir\sigma}(\omega) \equiv \pi t^2 \rho_0 \equiv \Gamma$  (we set  $\Gamma = 1$ ).

In order to determine the Green's function, we use the equation of motion method (EOM). To truncate higher order Green's functions, the self-consistent decoupling procedure proposed by Lacroix is used [8]. In the limit  $(U, U_1) \rightarrow \infty$  one gets

$$G_{i\sigma, i\sigma}^r(\omega) = \frac{1 - \sum_{l \in \Omega} (n_l - H_l(\omega))}{\omega - E_{i\sigma} - \Sigma_0 + \sum_{l \in \Omega} (F_l(\omega) - 2\Sigma_0 H_l(\omega))} \quad (3)$$

where  $\Sigma_0 = -i\Gamma$  is the self-energy for the non-interacting QD due to tunnelling of the  $i\sigma$  electron,  $\Omega = \{(\bar{i}, \sigma), (i, -\sigma), (\bar{i}, -\sigma)\}$ ,  $(\bar{1} = 2, \bar{2} = 1)$ ,  $n_l = (n_{lL} + n_{lR})/2$ , and

$$F_l(\omega) = \Gamma \sum_{\alpha} \int_{-D}^D \frac{d\omega'}{\pi} \frac{f_{\alpha}(\omega')}{\omega' - \omega - i0^+}, \quad H_l(\omega) = \Gamma \sum_{\alpha} \int_{-D}^D \frac{d\omega'}{\pi} \frac{f_{\alpha}(\omega') G_{l,l}^a(\omega')}{\omega' - \omega - i0^+} \quad (4)$$

### 3. Numerical results and discussion

For the considered strong inter and intra-dot interactions, the Kondo effect has two possible sources, spin and orbital degeneracies. For  $h = 0$  and  $\Delta E = \varepsilon_2 - \varepsilon_1 = 0$ , the spin and charge degrees of freedom of the DQD are totally entangled and the state possesses  $SU(4)$  symmetry. This corresponds to the highest conductance (DC) in Fig. 1 (VSD,  $h \approx 0$ ), where VSD denotes bias voltage.

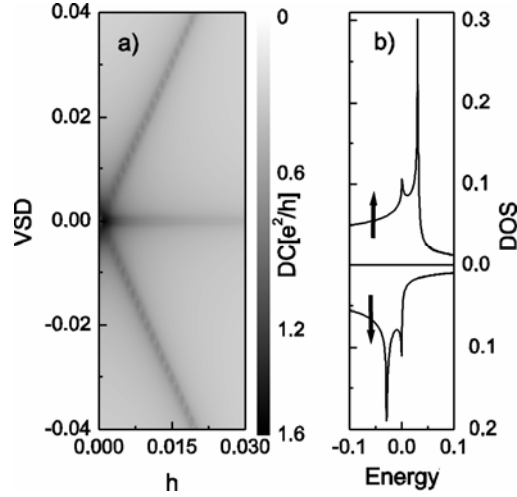


Fig. 1. Differential conductance vs. voltage bias and magnetic field for  $\varepsilon_1 = -4$ ,  $\Delta E = 0$  and  $g_1 = g_2 = 1$  (a) and many-body structure of the DOS for zero bias voltage and  $h = 0.015$  (b)

A magnetic field lifts the spin degeneracy. The observed high conductance line for zero bias ( $VSD = 0$ ) is due to orbital Kondo effects occurring in both spin channels separately ( $\varepsilon_{1\sigma} = \varepsilon_{2\sigma}$ ,  $2^*SU(2)$ ). In the many-body structure of the density of states (DOS), apart from the Kondo peaks there are also satellite peaks, located above and below the Fermi level at positions determined by the magnetic field (Fig. 1b). They result from the tunnelling processes, which link non-degenerate states. The satellites are responsible for the occurrence of high conductance lines with  $VSD = \pm 2h$ . Figure 2 presents the polarization of conductance in the considered case. The polarization of conductance is defined as  $PC = (\sigma_+ - \sigma_-) / (\sigma_+ + \sigma_-)$ , where  $\sigma$  denotes the differential conductance.

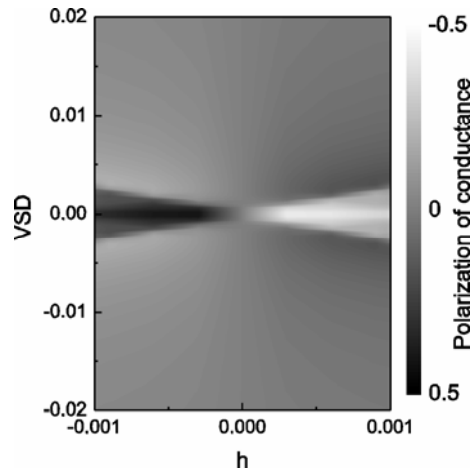


Fig. 2. Polarization of conductance vs. voltage bias and magnetic field for the same parameters as in Fig. 1

Opposite polarizations are observed close to the zero bias line for opposite field orientations. This can be used for spin filtering, what has been discussed earlier by Borda et al. [8]. The spin asymmetry of the current results from the Zeeman splitting of the bare energy levels. In the following, we discuss a different spin-filtering scenario. By an appropriate tuning of gate voltages, orbital degeneracy can be recovered for only one spin channel.

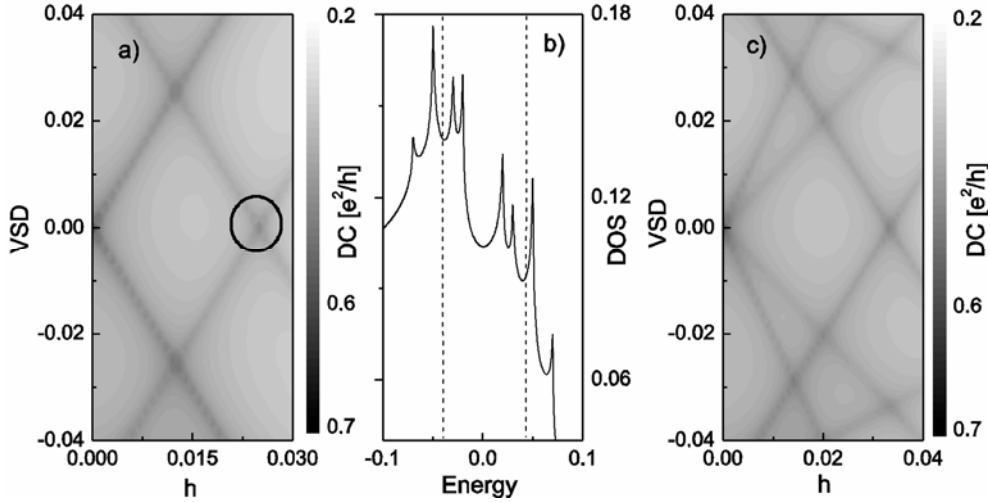


Fig. 3. Differential conductance for  $\Delta E = 0.05$ ,  $g_1 = \pm g_2 = 1$  (a) and  $g_1 = 1$ ,  $g_2 = 0.5$  (c).  
Fig. 3b presents the many-body structure for zero bias and  $h = 0.01$

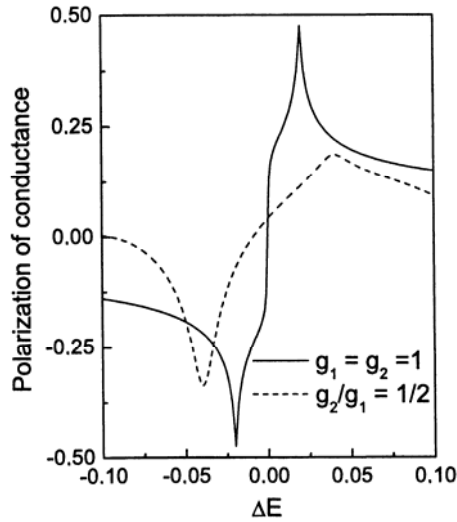


Fig. 4. Polarization of conductance vs. gate voltage ( $\Delta E$ )  
for  $g_1 = -g_2 = 1$  (solid line) and  $g_1 = 1$ ,  
 $g_2 = 0.5$  (dashed line), with  $\varepsilon_1 = -4$ ,  $h = 0.01$

Figure 3a presents the differential conductance of a DQD for  $\Delta E \neq 0$ . The high transparency region (VSD,  $h \approx 0$ ) corresponds to the spin Kondo effect at the dots ( $\varepsilon_{i+} = \varepsilon_{i-}$ ,  $2^*SU(2)$ ). The enhanced conductance in this region, marked by the dark circle, is due to the orbital Kondo effect ( $\varepsilon_{1+} = \varepsilon_{2-}$  for  $g_1 = g_2$ , or  $\varepsilon_{1+} = \varepsilon_{2+}$  for  $g_1 = -g_2$ ). The orbital degeneracy for the same spin orientation can be used for spin filtering. Reversing the spin polarization of conductance can be induced either by a change in gate voltage or by reversing the magnetic field (Fig. 4). For  $g_1 = -g_2$ , an antisymmetric change in polarization is evident (bipolar spin filter). The four high-conductance lines visible in Fig. 3a occur in regions where the satellite peaks of the DOS, located at  $E = \pm 2h \pm \Delta E$ , enter the region between the Fermi levels of the leads from opposite sides (Fig. 3b). This happens for bias voltages equal to the positions of the satellite peaks. The peaks in the DOS outside the energy region marked by the dashed vertical lines in Fig 3b do not play any role in the conductance for the voltage range of Fig 3a. DC for a more general case ( $g_1 \neq g_2$ ) is presented in Fig. 3c. The number of high transparency lines increases due to the lack of dot spin symmetry or antisymmetry. The tunnelling processes within the  $\{\varepsilon_{i\sigma}\}$  manifold are specified in this case by a larger number of characteristic energies.

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