Transport characteristics of ferromagnetic single -electron transistors with non-collinear magnetizations

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Theoretical analysis of spin-polarized transport through a ferromagnetic single-electron transistor (FM SET) has been carried out in the sequential tunnelling regime. Two external electrodes and the central part (island) of the device are assumed to be ferromagnetic, with the corresponding magnetizations being generally non-collinear. The transport properties of the FM SET are analysed within the master equation approach with the respective transition rates determined from the Fermi golden rule. It is assumed that spin relaxation processes on the island are sufficiently fast to neglect spin accumulation. It is shown that electric current and tunnel magnetoresistance strongly depend on the magnetic configuration of the device. Transport characteristics of symmetrical and asymmetrical structures have been calculated as a function of bias and gate voltages.

Key words: ferromagnetic single-electron transistor; spin-polarized transport; tunnel magnetoresistance

1. Introduction

There is a general need to produce smaller and faster processors, smaller and more voluminous memories, and smaller and more sensitive sensors. It seems that spintronics fulfils such expectations and demands. Moreover, in the nanometer scale we have to consider the increasing importance of interactions between electrons, so that the role of electron spin becomes important as well. The problem of spin-polarized transport in ferromagnetic single-electron transistors (FM SETs) with quantum dots or metallic grains [1–4] has been addressed only recently, particularly for non-collinear magnetic moments of the external electrodes [5]. In Ref. [5], a situation with one nonmagnetic and one magnetic electrode was considered, and it was shown that transport characteristics strongly depend on the angle between the magnetic moments of the lead and island.

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In this paper, we present the results of our theoretical analysis of spin-polarized electronic transport in a ferromagnetic single-electron transistor, whose all three electrodes, i.e. two external leads and the central part (island), are ferromagnetic, with the corresponding magnetizations being generally non-collinear, but oriented in a common plane. The angle α (β) between the angular spin moments of the right (left) lead and the island is arbitrary (see Fig. 1). Apart from this, we assume that an external gate voltage is applied to the island. Our objective is to analyse the dependence of electric current, differential conductance, and tunnel magnetoresistance (TMR) on the angles between magnetizations and on the transport and gate voltages.

Transport properties are analysed within the master equation approach, with the corresponding transition rates determined from the Fermi golden rule. In this analysis, we take into account only sequential tunnelling processes – with lowest-order perturbation theory [6]. It is also assumed that the spin relaxation on the island is sufficiently fast to neglect spin accumulation. We have analysed numerically the electric current flowing through the device, the corresponding differential conductance, and the resulting tunnel magnetoresistance for different magnetic configurations of the device. From this analysis it follows that all transport characteristics strongly depend on the angles α and β .

2. Model and theoretical description

A schematic diagram of the analysed ferromagnetic single-electron transistor is shown in Fig. 1. The device under consideration consists of three electrodes made of the same ferromagnetic material. When a sufficiently high bias voltage is applied to the device, electrons can tunnel through the barriers sequentially one by one, giving rise to electric current. When the charging energy is considerably larger than the thermal energy, charging effects become observable and the current–voltage characteristics display a typical Coulomb staircase with a Coulomb blockade in the small bias regime. Apart from this, Coulomb oscillations in the electric current occur with increasing gate voltage. These effects lead to an oscillatory behaviour of TMR with increasing bias voltage.



Fig. 1. A schematic diagram of the ferromagnetic single-electron transistor. The arrows indicate the spin moments of the island and the two external electrodes

In our considerations, we take into account only sequential tunnelling processes and assume that the contributions of higher-order processes are small compared to the first-order ones. This is justifiable when the barrier resistances significantly exceed quantum resistance, $R_j >> R_q = h/e^2$, j = 1, 2, which implies that the charge on the island is well localized and that orthodox tunnelling theory is applicable [6]. The resistances of the tunnel junctions depend on the tunnel matrix elements between the corresponding states of the left (labelled with 1) and right (labelled with 2) electrodes and the island, which in turn depend on the angle between magnetizations. Furthermore, we take into account only tunnelling processes that conserve electron spin when the magnetic moments of the leads are collinear. Apart from this, we assume that spin relaxation time on the island is shorter than the time between the two successive tunnelling events. This means that a non-equilibrium magnetic moment (spin accumulation) cannot build up on the island. Moreover, the island is assumed to be relatively large, so that quantization effects of the corresponding energy levels can be neglected.

The rate of electron tunnelling from the spin majority/minority (+/-) electron bands of the first (left) electrode to the spin majority/minority (+/-) electron channels of the island can be expressed in terms of the Fermi golden rule as:

$$\Gamma_{1 \to i}^{\pm \pm} = \frac{2\pi}{\hbar} \left| \left\langle \Psi_{i\pm} \right| H_T \left| \Psi_{i\pm} \right\rangle \right|^2 \delta(E_f - E_i) \tag{1}$$

where $| \Psi_{l\pm} \rangle$ and $| \Psi_{il\pm} \rangle$ are the wave functions of spin-majority (spin-minority) electrons in the first electrode and island, respectively, E_i and E_f are the initial and final energies of the whole system, and H_T is the tunnelling Hamiltonian. These wave functions are written in the respective local reference systems (with the local quantization axes determined by local spin moments). When a bias voltage V is applied to the device, the tunnelling rate from the first electrode to the island, already occupied by n excess electrons, can be written in the form:

$$\Gamma_{1 \to i}^{++(--)}(n, V) = \Gamma_{1 \to i}^{p,+(-)}(n, V) \cos^2 \frac{\beta}{2}$$
(2a)

$$\Gamma_{1 \to i}^{+-(-+)}(n,V) = \Gamma_{1 \to i}^{ap}(n,V) \sin^2 \frac{\beta}{2}$$
(2b)

with $\Gamma_{1\to i}^{p,+(-)}(n,V)$ denoting the tunnelling rate of spin majority (+) and spin minority (-) electrons in the parallel magnetic configuration. Similarly, $\Gamma_{1\to i}^{ap}(n,V)$ is the corresponding tunnelling rate in the antiparallel configuration. Note that $\Gamma_{1\to i}^{+-}(n,V) = \Gamma_{1\to i}^{-+}(n,V)$, which is a consequence of the assumption that all electrodes and the island are made of the same ferromagnetic material.

The rate $\Gamma_{1\to i}^{p+(-)}(n,V)$ is given by the formula:

$$\Gamma_{1 \to i}^{p,+(-)}(n,V) = \frac{1}{e^2 R_1^{p,+(-)}} \frac{\Delta E_1(n,V)}{\exp[\Delta E_1(n,V)/k_B T] - 1}$$
(3)

where *e* is the electron charge (e > 0), $R_1^{p,+(-)}$ denotes the spin-dependent resistance of the left junction in the parallel configuration, and k_BT is the thermal energy. Here, $\Delta E_1(n,V)$ describes a change in the electrostatic energy of the system caused by the respective tunnelling event. A similar expression also holds for $\Gamma_{1\to i}^{ap}(n,V)$, but with $R_1^{p,+(-)}$ replaced by R_1^{ap} . Since both ferromagnetic electrodes and the island are assumed to be made of the same material, the resistance R_1^{ap} is independent of electron spin. The rates of tunnelling from the island back to the first electrode and also of tunnelling through the second junction can be derived in a similar way.

In order to calculate the electric current flowing through the system in a stationary state, we take into account the fact that in a steady-state the net transition rate between charge states with n and n+1 excess electrons on the island is equal to zero [7]

$$P(n,V)\left|\sum_{\sigma=+,-\sigma'=+,-}\sum_{\sigma=+,-}\left[\Gamma_{1\to i}^{\sigma'\sigma}(n,V) + \Gamma_{2\to i}^{\sigma'\sigma}(n,V)\right]\right|$$

$$= P(n+1,V)\left[\sum_{\sigma=+,-\sigma'=+,-}\sum_{\sigma=+,-}\left[\Gamma_{i\to 1}^{\sigma\sigma'}(n+1,V) + \Gamma_{i\to 2}^{\sigma\sigma'}(n+1,V)\right]\right]$$
(4)

From this, one can determine the probability P(n,V) of finding the island in a state with *n* excess electrons when a bias voltage *V* is applied to the system.

Finally, the electric current flowing through the system from left to right can be calculated from the following formula:

$$I(V) = -e \sum_{\sigma=+,-} \sum_{\sigma'=+,-} \sum_{n=-\infty}^{\infty} \left[\Gamma_{1\to i}^{\sigma'\sigma}(n,V) - \Gamma_{i\to 1}^{\sigma\sigma'}(n,V) \right] P(n,V)$$
(5)

Equation (5) corresponds to the current flowing through the first junction, which in the stationary limit is equal to the current flowing through the second junction, and thus through the device.

3. Numerical results and discussion

Equation (5) can be used to calculate the tunnelling current for any magnetic configuration. The tunnel magnetoresistance can be described by the ratio [8]

$$TMR = \frac{I(\alpha = 0, \beta = 0)}{I(\alpha, \beta)} - 1$$
(6)

where $I(\alpha,\beta)$ is the current flowing when the angle between angular spin moments of the right electrode and island is equal to α and the angle between angular spin moments of the left electrode and island is β ($\alpha = 0$ and $\beta = 0$ corresponds to the parallel



Fig. 2. The bias dependence of electric current (a), differential conductance (b), and TMR (c) in a FM SET with non-collinear magnetizations for indicated values of the angles between angular spin moments. The parameters used for numerical calculations: T = 4.2 K, $C_1 = C_2 = C_g = 1$ aF, $V_g = 0$. Bias voltage applied symmetrically, the following resistances assumed: $R_1^{p,+} = 0.5$ MΩ, $R_1^{p,-} = 0.1$ MΩ, $R_2^{p,+} = 25$ MΩ, $R_2^{p,-} = 5$ MΩ and $R_i^{ap} = \sqrt{R_i^{p,+}R_i^{p,-}}$, i = 1, 2. The gate voltage dependences of electric current (d), the derivative dI/dV_g (e), and TMR (f) have been calculated for the indicated values of the angles α and β and for V = 40 mV

configuration). In Figure 2, we present the results of our numerical calculations of electric current flowing through the system and TMR as a function of the bias and gate voltages for several values of the angles ($\alpha = \beta$) between magnetizations. We have also calculated differential conductance as a function of the bias voltage and current sensitivity to the gate voltage, dI/dV_g , as a function of V_g .

For all magnetic configurations, the dependence of the electric current on the bias voltage is non-linear and presents characteristic Coulomb staircases, as is shown explicitly in Fig. 2a for three values of the angle between magnetic moments. The dependence of differential conductance on the bias voltage shows peaks corresponding to Coulomb steps in the current-voltage curves (Fig. 2b). In turn, the difference in currents flowing in two magnetic configurations – parallel ($\alpha = \beta = 0$) and non-collinear ($\alpha, \beta \neq 0$) – gives rise to a nonzero tunnel magnetoresistance, as shown in Fig. 2c. For an arbitrary magnetic configuration, TMR is modulated by charging effects which leads to characteristic dips (Fig. 2c) at bias voltages corresponding to Coulomb steps in the current-voltage characteristics of Fig. 2a. The amplitude of these dips decreases with increasing bias voltage. The gate voltage dependence of the electric current presents characteristic Coulomb oscillations (Fig. 2d). The corresponding derivative of electric current with respect to the gate voltage is shown in Fig. 2e. Finally, the Coulomb oscillations in electric current lead to the periodic behaviour of TMR with increasing gate voltage, as shown in Fig. 2f. Thus, the electric current and TMR effect strongly depend on the angles between the magnetic moments of the leads and island.

4. Conclusions

We have studied the transport characteristics of a ferromagnetic single-electron transistor. The device consists of a ferromagnetic island and two ferromagnetic external electrodes, whose magnetic moments can be oriented arbitrarily. We have calculated the current flowing through such a device, differential conductance, and the corresponding tunnel magnetoresistance. The bias dependence of electric current reveals characteristic Coulomb staircases, while the TMR effect shows characteristic dips. Furthermore, the current flowing through the system as well as the tunnel magnetoresistance also strongly depend on the gate voltage.

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